Exercícios – Combinações Lineares

Exercícios

1. Determine os espaços gerados pelos conjuntos de vetores a seguir:

   a. $A = \{(-2,1,-2), (-1,0,-2)\}$

   $$\begin{bmatrix} 1 & 0 & y \\ 0 & 1 & -x - 2y \\ 0 & 0 & -2x - 2y + z \end{bmatrix}$$

   $$S = \{(x,y,z) | z = 2x + 2y\}$$

   b. $A = \{(-1,1,2), (0,-1,1)\}$

   $$\begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -x - y \\ 0 & 0 & 3x + y + z \end{bmatrix}$$

   $$S = \{(x,y,z) | z = -3x - y\}$$

   c. $A = \{(0,1,-2,-2), (-2,-2,-1,10)\}$

   $$\begin{bmatrix} 1 & 0 & -x + y \\ 0 & 1 & -0.5x \\ 0 & 0 & 2.5x + 2y + z \\ 0 & 0 & 3x + 2y + t \end{bmatrix}$$

   $$S = \{(x,y,z,t) |\ z = -2.5x - 2y, t = -3x - 2y\}$$

   d. $A = \{(1,3), (2,4), (-1,-5)\}$

   $$\begin{bmatrix} 1 & 0 & -3 & -2x + y \\ 0 & 1 & 1 & 1.5x - 0.5y \end{bmatrix}$$

   $$S = \{(x,y) |\ x, y \in \mathbb{R}\}$$

   e. $A = \{(2,-2,-4), (1,-2,-3), (1,0,-1), (0,2,2)\}$

   $$\begin{bmatrix} 1 & 1 & 0 & -1 & -0.5y \\ 0 & 0 & 0 & 0 & x - y + z \\ 0 & -1 & 1 & 2 & x + y \end{bmatrix}$$

   $$S = \{(x,y,z) | z = -x + y\}$$

2. interprete as matrizes escalonadas reduzidas de forma a especificar o subespaço vetorial gerado.

   a. $A = \{(1,5), (0,0)\}$

   $$\begin{bmatrix} 1 & 0 & x \\ 0 & 0 & -5x + y \end{bmatrix}$$

$$S = \{(x, y) | y = 5x\}$$

b. $A = \{(1, -1, -1)\}$

$$\begin{bmatrix} 1 & x \\ 0 & x + y \\ 0 & x + z \end{bmatrix}$$

$$S = \{(x, y, z) | y = -x, z = -x\}$$

c. $A = \{(0,0,-1,0), (2,-2,0,6)\}$

$$\begin{bmatrix} 0 & 0 & x + y \\ 0 & 1 & -0.5y \\ -1 & 0 & z \\ 0 & 0 & 3y + t \end{bmatrix}$$

$$S = \{(x, y, z, t) | x = -y, t = -3y\}$$

d. $A = \{(1, -1,1), (-1,2, -4)\}$

$$\begin{bmatrix} 1 & 0 & 2x + y \\ 0 & 1 & x + y \\ 0 & 0 & 2x + 3y + z \end{bmatrix}$$

$$S = \{(x, y, z) | z = -2x - 3y\}$$

e. $A = \{(-2,2, -8), (-2, -1, -2), (0,1, -2)\}$

$$\begin{bmatrix} 1 & 1 & 0 & -0.5x \\ 0 & 0 & 0 & -2x + 2y + z \\ 0 & -3 & 1 & x + y \end{bmatrix}$$

$$S = \{(x, y, z) | z = 2x - 2y\}$$

f. $A = \{(1,0, -1), (2,0,2), (1, -2,1), (2,0,2)\}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0.5x - 0.5z \\ 0 & 1 & 0 & 1 & 0.25x + 0.25y + 0.25z \\ 0 & 0 & 1 & 0 & -0.5y \end{bmatrix}$$

$$S = \{(x, y, z) | x, y, z \in \mathbb{R}\}$$