

restart;

em paralelo: soma-se

$$c[1] := \frac{\text{epsilon}[0] \cdot (A[1] - a[1])}{d[1]} + \frac{\text{epsilon}[0] \cdot a[1] \cdot \kappa[1]}{d[1]};$$
$$\frac{\epsilon_0 (A_1 - a_1)}{d_1} + \frac{\epsilon_0 a_1 \kappa_1}{d_1} \quad (1)$$

em série: o inverso da soma dos inversos

$$c[2] := \frac{1}{\frac{1}{\text{epsilon}[0] \cdot A[2]} + \frac{1}{\text{epsilon}[0] \cdot A[2] \cdot \kappa[2]}};$$
$$\frac{1}{\frac{1}{\epsilon_0 A_2} + \frac{1}{\epsilon_0 A_2 \kappa_2}}$$
$$\frac{d_2 - a_2}{\epsilon_0 A_2} + \frac{a_2}{\epsilon_0 A_2 \kappa_2} \quad (2)$$

$$c[3] := \frac{\text{epsilon}[0] \cdot A[3]}{d[3]};$$
$$\frac{\epsilon_0 A_3}{d_3} \quad (3)$$

em paralelo

$$\text{ceq}[1] := c[2] + c[3];$$
$$\frac{1}{\frac{d_2 - a_2}{\epsilon_0 A_2} + \frac{a_2}{\epsilon_0 A_2 \kappa_2}} + \frac{\epsilon_0 A_3}{d_3} \quad (4)$$

em série, que é a capacitância equivalente do circuito com apenas um capacitor

$$\text{ceq}[2] := \frac{1}{\frac{1}{c[1]} + \frac{1}{\text{ceq}[1]}};$$
$$\frac{1}{\frac{1}{\frac{\epsilon_0 (A_1 - a_1)}{d_1} + \frac{\epsilon_0 a_1 \kappa_1}{d_1}} + \frac{1}{\frac{1}{\frac{d_2 - a_2}{\epsilon_0 A_2} + \frac{a_2}{\epsilon_0 A_2 \kappa_2}} + \frac{\epsilon_0 A_3}{d_3}}} \quad (5)$$

"simplificando"

simplify(ceq[2]);

$$-(\epsilon_0 (a_1 \kappa_1 + A_1 - a_1) (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2)) / (a_1 a_2 d_3 \kappa_1 \kappa_2$$
$$- a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2$$
$$- a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3) \quad (6)$$

carga no capacitor equivalente 2, que é do circuito com apenas um capacitor

$$qeq[2] := \text{simplify}(ceq[2] \cdot V[0]);$$

$$-(\varepsilon_0 (a_1 \kappa_1 + A_1 - a_1) (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2) V_0) / (a_1 a_2 d_3 \kappa_1 \kappa_2 - a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2 - a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3) \quad (7)$$

a carga em cada capacitor, $c[1]$ e $ceq[1]$, são iguais às cargas de $ceq[2]$. Assim, a tensão em $c[1]$ é:

$$q[1] := qeq[2];$$

$$V[1] := \text{simplify}\left(\frac{q[1]}{c[1]}\right);$$

$$-(d_1 V_0 (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2)) / (a_1 a_2 d_3 \kappa_1 \kappa_2 - a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2 - a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3) \quad (8)$$

a tensão em $c[2]$ e $c[3]$ são as mesmas, e iguais a $V[0]-V[1]$

$$V[2] := V[0] - V[1];$$

$$V_0 + (d_1 V_0 (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2)) / (a_1 a_2 d_3 \kappa_1 \kappa_2 - a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2 - a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3) \quad (9)$$

$$q[2] := V[2] \cdot c[2];$$

$$\frac{1}{\frac{d_2 - a_2}{\varepsilon_0 A_2} + \frac{a_2}{\varepsilon_0 A_2 \kappa_2}} (V_0 + (d_1 V_0 (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2))) / (a_1 a_2 d_3 \kappa_1 \kappa_2 - a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2 - a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3) \quad (10)$$

$$q[3] := V[2] \cdot c[3];$$

$$\frac{1}{d_3} ((V_0 + (d_1 V_0 (A_2 d_3 \kappa_2 - A_3 a_2 \kappa_2 + A_3 d_2 \kappa_2 + A_3 a_2))) / (a_1 a_2 d_3 \kappa_1 \kappa_2 - a_1 d_2 d_3 \kappa_1 \kappa_2 + A_1 a_2 d_3 \kappa_2 - A_1 d_2 d_3 \kappa_2 - A_2 d_1 d_3 \kappa_2 + A_3 a_2 d_1 \kappa_2 - A_3 d_1 d_2 \kappa_2 - a_1 a_2 d_3 \kappa_1 - a_1 a_2 d_3 \kappa_2 + a_1 d_2 d_3 \kappa_2 - A_1 a_2 d_3 - A_3 a_2 d_1 + a_1 a_2 d_3)) \varepsilon_0 A_3) \quad (11)$$

ratificação de que $q[1]=q[2]+q[3]$

$$\text{simplify}(q[2] + q[3] - q[1]);$$

0

(12)

$$A[1] := 10^{-4}; A[2] := A[1]; A[3] := A[1];$$

$$d[1] := 4 \cdot 10^{-6}; d[2] := d[1]; d[3] := d[1];$$

$$a[1] := 2 \cdot 10^{-5}; a[2] := 10^{-6};$$

$$\text{kappa}[1] := 5; \text{kappa}[2] := 10;$$

$$\begin{aligned}
&\text{epsilon}[0] := 8.85 \cdot 10^{-12}; \\
&V[0] := 120; \\
&\# \text{ carga } q[1] \\
&\text{evalf}(q[1]);
\end{aligned}
\qquad
2.675938486 \cdot 10^{-8}
\qquad
(13)$$

$$\begin{aligned}
&\# \text{ carga } q[2] \\
&\text{evalf}(q[2]);
\end{aligned}
\qquad
1.507570978 \cdot 10^{-8}
\qquad
(14)$$

$$\begin{aligned}
&\# \text{ carga } q[3] \\
&\text{evalf}(q[3]);
\end{aligned}
\qquad
1.168367508 \cdot 10^{-8}
\qquad
(15)$$

$$\begin{aligned}
&\# \text{ tensão } V[1] \\
&\text{evalf}(V[1]);
\end{aligned}
\qquad
67.19242902
\qquad
(16)$$

$$\begin{aligned}
&\# \text{ tensão } V[2] \text{ e } V[3]=V[2] \\
&\text{evalf}(V[2]);
\end{aligned}
\qquad
52.80757098
\qquad
(17)$$

$$\begin{aligned}
&\# \text{ energia } c[1] \\
&U[1] := \frac{c[1] \cdot V[1]^2}{2};
\end{aligned}
\qquad
8.990140340 \cdot 10^{-7}
\qquad
(18)$$

$$\begin{aligned}
&\# \text{ energia } c[2] \\
&U[2] := \frac{c[2] \cdot V[2]^2}{2};
\end{aligned}
\qquad
3.980558070 \cdot 10^{-7}
\qquad
(19)$$

$$\begin{aligned}
&\# \text{ energia } c[3] \\
&U[3] := \frac{c[3] \cdot V[2]^2}{2};
\end{aligned}
\qquad
3.084932505 \cdot 10^{-7}
\qquad
(20)$$